A Multi-objective Optimization Approach to Balancing Utility and Equity in Location-allocation Problems

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ABSTRACT: Location-allocation problems, widely applied in managing public resources and services, often prioritize utility by optimizing overall efficiency, cost, or service coverage when determining facility locations. However, the focus on utility can be at the expense of an equitable distribution of benefits among different population subgroups. This paper introduces a multi-objective optimization approach to balancing utility and equity. We apply this approach to environmental sensor placement in Chicago, which indicates its effectiveness in enhancing equitable sensor coverage and fostering justice in sustainable smart cities.

KEYWORDS: environmental justice, inequality, Maximal Covering Location Problem (MCLP), sensor networks, spatial optimization

Introduction

Location–allocation problems are a class of optimization problems that involve determining optimal facility locations and resource allocation to these facilities based on certain criteria (R. L. Church, 2005). Classical problems often prioritize *utility* by seeking to optimize some measure of overall efficiency, cost, or service coverage. A prominent example is the Maximal Covering Location Problem (MCLP), with the objective to maximize the *total* coverage of demand nodes given a limited number of facilities (R. Church & ReVelle, 1974). However, the focus on utility may not always ensure and, in fact, often conflicts with *equity*: certain individuals or demographic groups may not receive a fair share of benefits from the facilities (Meyer, 2008). This can be particularly pertinent considering the broad applications of location-allocation problems in managing public resources and services.

Research has advocated for incorporating equity considerations into the formulation of location-allocation problems, with a predominant focus on equal accessibility (Li et al., 2017; C. H. Wang & Chen, 2021). F. Wang & Tang (2013), for example, introduce the Maximal Accessibility Equality Problem (MAEP) that minimizes the variance in accessibility index. While useful in promoting equal spatial opportunities for reaching facilities, the focus on accessibility does not address disparities in the benefits provided to different population subgroups. An alternative approach is presented in Robinson et al. (2022), which uses multi-objective optimization to balance coverage across a selected number of demographic subgroups. However, this approach models the coverage of each subgroup as a separate objective, potentially leading to computational intensity as the number of subgroups increases.

The purpose of this paper is to introduce a multi-objective optimization approach to balancing utility and equity of coverage among different population subgroups, with a specific focus on the MCLP–a widely applied location-allocation problem. Different from Robinson et al. (2022), we use a measure to summarize inequality among all subgroups, eliminating the need for an extensive number of defined objectives. Our method is evaluated through a case study of environmental sensor placement in Chicago, which demonstrates its effectiveness in providing more equitable solutions for sensor coverage compared to those achievable through the MCLP.

Methods

We extend the classical MCLP to incorporate an additional objective of minimizing inequality through multi-objective optimization. The following parameters and variables define the optimization model.

Input parameters:

I = set of all demand nodes (indexed by i),
J = set of all candidate facility sites (indexed by j),
K = set of all population subgroups (indexed by k),
p = number of facilities to locate,
w_{ik} = demand of subgroup k at node i,
a_{ij} = coverage of node i by candidate site j.

Decision variables:

$$x_j = \begin{cases} 1, & \text{if candidate site } j \text{ is selected,} \\ 0, & \text{otherwise,} \end{cases}$$
y_i = coverage of node i.

Coverage modeling

The coverage parameter a_{ij} is a critical input, adaptable to different applications through discrete and continuous models. Let d_{ij} be the Euclidean distance between node i and candidate site j, s be the maximum effective coverage distance of a site, and t be a distance decay constant. In the discrete model, $a_{ij} = 1$ if $d_{ij} \leq s$, and 0 otherwise, indicating binary coverage. The continuous model, $a_{ij} = e^{-td_{ij}/s}$, accommodates partial coverage with a smooth transition based on distance.

Measures of inequality

There are different ways of examining social inequality in the literature. This study explores three commonly used measures—relative range, variance, and Theil index—which will then be used as objective functions in our optimization model. To compute these measures of inequality, we define the percent coverage for each subgroup k as

$$r_k = \frac{\sum_{i\in I} w_{ik} y_i}{\sum_{i\in I} w_{ik}}, \quad (1)$$

which illustrates the extent to which the sitting facilities benefit a subgroup, normalized by its population. The average percent coverage across all subgroups is denoted as $\bar{r}$. The number of subgroups is denoted as $|K|$. The three measures of inequality considered are described below.

(1) The *relative range* is the maximum difference between the percent coverage of any two subgroups ($\max_{k \in K} r_k - \min_{k \in K} r_k$), normalized by the average coverage ($\bar{r}$):

$$E = \frac{\max_{k \in K} r_k - \min_{k \in K} r_k}{\bar{r}} . \qquad (2)$$

(2) The *variance* is the average of the squared differences between each subgroup's percent coverage and the average coverage ($\bar{r}$):

$$E = \frac{1}{|K|} \sum_{k \in K} (r_k - \bar{r})^2 . \qquad (3)$$

(3) The *Theil index*, introduced by economist Henri Theil, is one of a family of entropy measures used to quantify various aspects of inequality (Theil, 1967):

$$E = \frac{1}{|K|} \sum_{k \in K} \frac{r_k}{\bar{r}} \ln \frac{r_k}{\bar{r}} . \qquad (4)$$

All three measures of inequality have a minimum value of 0 that indicates complete equity. In other words, minimizing the values of these measures corresponds to maximizing equity.

Multi-objective optimization

An optimization model is formulated to maximize both utility and equity by maximizing the total coverage across all subgroups (Eq. 5) while minimizing a measure of inequality (Eq. 6):

$$\begin{array}{lll}
\max & U = \sum_{k \in K} \sum_{i \in I} w_{ik} y_i \ , & (5) \\
\min & E \ , & (6) \\
\text{subject to} & y_i = \max_{j \in J} a_{ij} x_j \quad \forall i \ , & (7) \\
 & \sum_{j \in J} x_j \leq p \ , & (8) \\
 & x_j \in \{0,1\} \quad \forall j \ , & (9) \\
 & 0 \leq y_i \leq 1 \quad \forall i \ . & (10)
\end{array}$$

Constraints in Eq. 7 state that, for each node *i*, the coverage is equal to the maximum coverage value across all candidate facility sites. This differs from the classic MCLP, where each node is considered either covered or not covered. In our formulation, we accommodate partial coverage, which aligns better with practical applications. Eq. 8 specifies that the total number of selected facility sites cannot exceed the specified number of facilities to be located. Constraints in Eqs. 9 and 10 specify the range of decision variables.

Solving this optimization problem is expected to yield a set of Pareto-optimal solutions, where no one objective can be improved without compromising the other. While, like many other location-allocation problems, it can be challenging to find optimal solutions using exact methods, this problem can be effectively solved with heuristic methods such as the Non-dominated Sorting Genetic Algorithm II (NSGA-II) (Deb & Jain, 2014), as applied in our subsequent computational experiments.

Experimental Results

We implement the proposed approach to identify the optimal locations for air quality sensors in Chicago. Sensor networks are pivotal components of sustainable smart cities, and Chicago has been actively involved in deploying environmental sensors through initiatives such as the Array-of-Things. Ensuring equitable sensor coverage is essential for promoting environmental justice and fostering an inclusive and green urban future.

Figure 1 illustrates the Pareto-optimal solutions resulting from our method, indicating reduced coverage inequality compared to those from the MCLP. Notably, when using continuous coverage, it is potential to derive a set of optimal solutions with more equitable coverage without degrading much of the utility (total coverage) achievable through the MCLP.

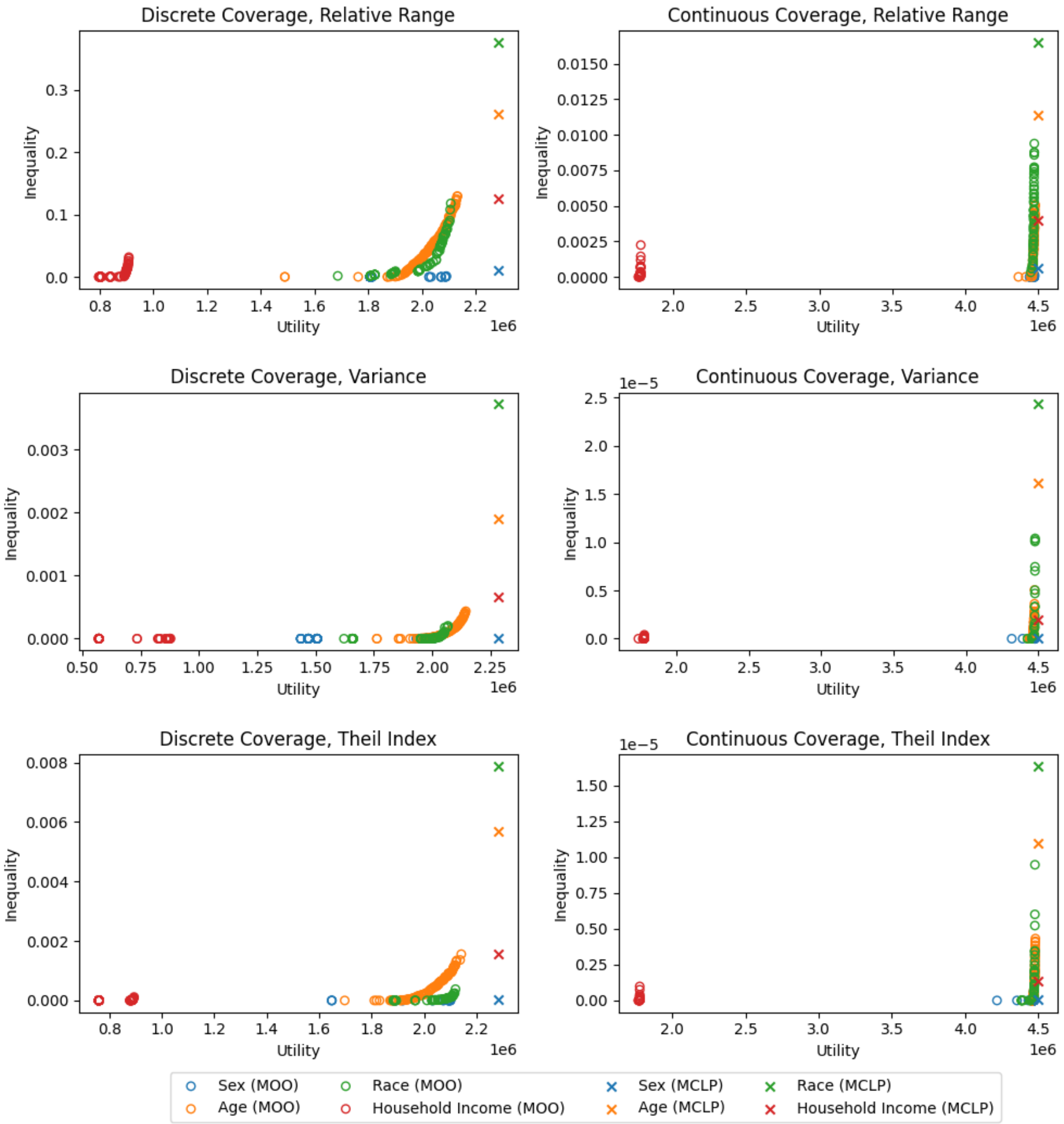

Figure 1: Optimal solutions resulting from MOO (multi-objective optimization) (marked by circle symbols) and the MCLP (marked by cross symbols).

Conclusions

This paper introduces a multi-objective optimization approach to balancing utility and equity in location modeling and demonstrates its effectiveness in selecting optimal locations for environmental sensors. While our focus is on the MCLP, the way of incorporating measures of inequality in model formulation can be extended to other location-allocation problems, such as the *p*-median problem.

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